

# Hawking radiation via Anomaly and Tunneling method from Unruh's and Canonical acoustic black hole

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We study the Hawking radiation from Unruh's and Canonical acoustic black hole from viewpoint of anomaly cancelation method developed by Robinson and Wilczek and by the simple and physically intuitive picture given by the tunneling mechanism.

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## I. INTRODUCTION

Hawking radiation is an important quantum effect of black hole physics. From the quantum point of view, black holes are not completely black and they can emit radiation with a temperature given by  $h/8\pi k_B GM$  [1], while from a classical point of view, nothing can escape from the black holes. Essentially this radiation is thermal and implies black hole could slowly evaporate emitting quantas. This radiation has been considered as main tools to understand the quantum nature of gravity. Hawking radiation shows universality properties and, also it is determined by universal properties of the event horizon. There are several approach to obtain the Hawking radiation, one of them, was development by Christensen and Fulling [2], who showed that Hawking radiation in (1+1)-dimensional Schwarzschild background metric can be derive from the trace anomaly or conformal anomaly, which arise from the renormalization of the quantized theory. On recently derivation, knowing as tunneling method, was proposed in Ref. [3, 4, 5, 6]. Due its simplicity this method has attracted a lot of attention and subsequent work, in the tunneling method, Ref. [3, 7, 8], one calculated the imaginary part of the action for the (classically forbidden) process of s-wave emission across the horizon using the null geodesic equation [3], which in turn is related to the Boltzmann factor for the emission at the Hawking temperature, while the complex path method was first proposed by K.Srinivasan and T.Padmanabhan [4] and subsequently developed by many authors [9, 10]. The method developed employs the Hamilton-Jacobi equations to obtain the particle classical action along with detailed balance of the ingoing and outgoing probability amplitudes. The Hamilton-Jacobi method has been applies to more complicated spacetimes [11] and to dynamical black holes [12, 13, 14]. However, in Ref. [15, 16], it was shown that the WKB/Tunneling method appeared to gave a temperature twice as large as the correct Hawking temperature. Recently, this problem has been solved in Ref. [17, 18], where it was shown that in contrast to normal quantum mechanical WKB/Tunneling problems that the tunneling probability received a contribution from the time coordinate upon crossing the horizon. By requiring canonical invariance of the tunneling amplitude and taking into account the temporal contribution one obtain the correct Hawking temperature. Another recent approach to the problem of Hawking radiation was developed by Robinson and Wilczek in Ref. [19], where the authors combine ideas of gravitational anomaly and the Hawking radiation of Schwarzschild type black holes in order to determinate Hawking radiation. This method has acquired a growing interest and has been applied for several geometries that described black holes [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35] all those references have showed the robustness of the anomaly gravitational methods for figure out the nature of universality, and quantum properties of Hawking radiation. We would like to note, this effect coming from the event horizon and therefore is a pure kinematical effect that occurs in any Lorenzian geometry independent of its dynamical content. This allow to consider analog models that mimic the properties of black hole physics. In this sense, the field of analog models of gravity allows in principle that the most processes of black hole physics can be studied in a laboratory. In particular the use of supersonic acoustics flows as an analogy to gravitating systems was for first the time proposed by Unruh [36] and with the works of Visser [37, 38, 39, 40, 41] has received an exponentially growing attention. In this article we computed the Hawking temperature for two acoustic geometry by the used of the method developed by Robinson and Wilczek and by the tunneling method via Hamilton-Jacobi ansatz.

The organization of the paper is as follows: In Sec. II, we specify the gravitational anomaly method developed by Robinson and Wilczek. In Sec. III, we discuss as emerged acoustic geometry and we apply the gravitational anomaly method to Unruh's sonic black hole. In Sec. IV, we determine the Hawking temperature for the Canonical acoustic black hole via Robinson-Wilczek method. In Sec. V, we find the Hawking temperature for both black holes via tunneling method and finally, we conclude in Sec. VI.

## II. ROBINSON-WILCZEK METHOD FOR THE SCHWARZSCHILD TYPE BLACK HOLE

Robinson and Wilczek in Ref.[19] have showed that the energy flux of the Hawking radiation can be fixed by the value of the gravitational anomaly at the horizon. Here the universality of Hawking radiation is connected to the universality of the gravitational anomaly. The starting point was considering a  $d$ -dimensional Schwarzschild type space-time with the metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{(d-2)}^2, \quad (1)$$

where  $d\Omega_{(d-2)}^2$  is the line element on the  $(d-2)$ -sphere and  $f(r)$  is dependent on the matter distribution. They considered the case where  $f(r)$  has exactly one positive, real root and all derivatives of  $f(r)$  are finite on the horizon. On the other hand, the contribution to effective action for the metric  $g_{\mu\nu}$  due to matter fields that interact with this metric is given by

$$W[g_{\mu\nu}] \equiv -i \ln \left( \int D[matter] e^{iS[matter, g_{\mu\nu}]} \right), \quad (2)$$

where  $S[matter, g_{\mu\nu}]$  is the classical action functional. Under general coordinate transformations the classical action  $S$  changes as

$$\delta_\lambda S = - \int d^d x \sqrt{-g} \lambda^\nu \nabla_\mu T_\nu^\mu, \quad (3)$$

where  $T_\nu^\mu$  is the energy-momentum tensor and  $\lambda$  is the variational parameter. The symmetry of the classical action (general covariance) requires that

$$\delta_\lambda S = 0 \Rightarrow \nabla_\mu T_\nu^\mu = 0. \quad (4)$$

In order to avoid problems of divergence associated with the Boulware vacuum, Robinson and Wilczek excluded the propagating modes along one light like direction. The price to pay for this assumption is that the effective quantum theory is chiral near the horizon and chiral theories contain gravitational anomalies [19]. In consequence, condition (3) is quantum mechanically violated due to the chiral anomaly. Therefore, a real energy-momentum flux it is needed as a compensating object. Main idea of the method, consist in the divergence of this flux will be canceled the anomaly at the horizon. General covariance of the full quantum theory requires  $\delta_\lambda W = 0$ . It is write as

$$-\delta_\lambda W = \int d^2 x \sqrt{-g} \lambda^\nu \nabla_\mu T_\nu^\mu, \quad (5)$$

where

$$T_\nu^\mu = T_{o\nu}^\mu \Theta_+ + T_{i\nu}^\mu \Theta_- + T_{\chi\nu}^\mu H, \quad (6)$$

$\Theta_\pm = \Theta(\pm r \mp r_H - \varepsilon)$  are scalar step functions and  $H = 1 - \Theta_+ - \Theta_-$ . The energy-momentum tensors  $T_{o\nu}^\mu$  and  $T_{i\nu}^\mu$  are covariantly conserved outside and inside the horizon, respectively. However,  $T_{\chi\nu}^\mu$  in (6) is not conserved due to the chiral anomaly at the horizon and this anomaly is purely time-like and given by [43]

$$\nabla_\mu T_{\chi\nu}^\mu \equiv A_\nu \equiv \frac{1}{\sqrt{-g}} \partial_\mu N_\nu^\mu, \quad (7)$$

where

$$N_\nu^\mu = \frac{1}{96\pi} \epsilon^{\beta\mu} \partial_\alpha \Gamma_{\nu\beta}^\alpha, \quad (8)$$

and  $\epsilon^{\beta\mu}$  is the two dimensional Levi-Civita tensor. Eq.(5) can be write as

$$\begin{aligned} -\delta_\lambda W &= \int d^2 x \sqrt{-g} \lambda^\nu \nabla_\mu \{ T_{\chi\nu}^\mu H + T_{o\nu}^\mu \Theta_+ + T_{i\nu}^\mu \Theta_- \}, \\ &= \int d^2 x \lambda^t \{ \partial_r (N_t^r H) + (T_{ot}^r - T_{\chi t}^r + N_t^r) \partial \Theta_+ + (T_{it}^r - T_{\chi t}^r + N_t^r) \partial \Theta_- \} \\ &\quad + \int d^2 x \lambda^r \{ (T_{or}^r - T_{\chi r}^r) \partial \Theta_+ + (T_{ir}^r - T_{\chi r}^r) \partial \Theta_- \}. \end{aligned} \quad (9)$$

Form of the  $T_\nu^\mu$  up to an arbitrary function of  $r$  and two constants of integration,  $K$  and  $Q$ , is given by

$$\begin{aligned} T_t^t &= -\frac{(K+Q)}{f} - \frac{B(r)}{f} - \frac{I(r)}{f} + T_\alpha^\alpha, \\ T_r^r &= \frac{(K+Q)}{f} + \frac{B(r)}{f} + \frac{I(r)}{f}, \\ T_t^r &= -K + C(r) = -f^2 T_r^t, \end{aligned} \quad (10)$$

where

$$B(r) = \int_{r_H}^r f(x) A_r(x) dx, \quad (11)$$

$$C(r) = \int_{r_H}^r A_t(x) dx, \quad (12)$$

$$I(r) = \frac{1}{2} \int_{r_H}^r T_\alpha^\alpha(x) f'(x) dx. \quad (13)$$

Here, was assumed that  $\frac{I}{f}|_{r_H} = \frac{1}{2} T_\alpha^\alpha|_{r_H}$  becomes finite, and

$$\lim_{(r-r_H) \rightarrow 0_-} \left(\frac{1}{f}\right) = - \lim_{(r-r_H) \rightarrow 0_+} \left(\frac{1}{f}\right). \quad (14)$$

In the limit  $\varepsilon \rightarrow 0$ , using (14) the variation (9) becomes

$$\begin{aligned} \delta_\lambda W &= \int d^2 x \lambda^t \{ [K_o - K_i] \delta(r - r_H) - \varepsilon [K_o + K_i - 2K_\chi - 2N_t^r] \partial \delta(r - r_H) + \dots \} \\ &\quad - \int d^2 x \lambda^r \left[ \frac{K_o + Q_o + K_i + Q_i - 2K_\chi - 2Q_\chi}{f} \right] \delta(r - r_H) \\ &\quad + \int d^2 x \lambda^r \varepsilon \left[ \frac{K_o + Q_o - K_i - Q_i}{f} \right] \partial \delta(r - r_H) + \dots, \end{aligned} \quad (15)$$

where

$$K_o = K_i = K_\chi + \Phi, \quad (16)$$

$$Q_o = Q_i = Q_\chi - \Phi, \quad (17)$$

and

$$\Phi = N_t^r|_{r_H}, \quad (18)$$

then total energy-momentum tensor reads,

$$T_\nu^\mu = T_{o\nu}^\mu \Theta_+ + T_{i\nu}^\mu \Theta_- + T_{\chi\nu}^\mu H, \quad (19)$$

in the limit  $\varepsilon \rightarrow 0$ , becomes

$$T_\nu^\mu = T_{c\nu}^\mu + T_{\Phi\nu}^\mu, \quad (20)$$

where  $T_{c\nu}^\mu$  is the conserved energy-momentum tensor without any quantum effects, and  $T_{\Phi\nu}^\mu$  is a conserved tensor with  $K = -Q = \Phi$ . Therefore, for a metric of the form (1) the components of  $N_\nu^\mu$  are

$$N_t^t = N_r^r = 0, \quad (21)$$

$$N_t^r = \frac{1}{192\pi} (f'^2 + f''f), \quad (22)$$

$$N_r^t = \frac{-1}{192\pi f^2} (f'^2 - f''f). \quad (23)$$

so

$$\Phi = N_t^r = \frac{1}{192\pi} f'^2(r_H). \quad (24)$$

In addition, it is well known that the surface gravity  $k$  in this case is given by

$$k = \frac{1}{2} \frac{\partial f}{\partial r} = \frac{1}{2} f'(r_H), \quad (25)$$

which implies the following Hawking temperature

$$T_H = \frac{k}{2\pi} = \frac{f'(r_H)}{4\pi}. \quad (26)$$

On the other hand, a beam of massless blackbody radiation moving in the positive  $r$  direction at a temperature  $T$  has a flux of the form

$$\Phi = \frac{\pi}{12} T_H^2. \quad (27)$$

Those results mean the flux required to cancel the gravitational anomaly at the horizon has a form equivalent to blackbody radiation with a temperature given by  $T = k/(2\pi)$  and this is exactly the Hawking temperature for this spacetime. Thus, the thermal flux required by black hole thermodynamics is able of canceling the anomaly.

### III. UNRUH'S ACOUSTIC BLACK HOLE

Acoustic geometry or the use of supersonic acoustics flows as an analogy to gravitating systems was for first time proposed by Unruh [36] and with the Visser's works [37, 38, 39, 40, 41] has received an exponentially growing attention. From the original Unruh's idea considering the motion of sound waves in the isentropic and convergent fluid flow. It is possible to obtain the Eulerian equation of motion from the action

$$S = - \int d^4x \left( \rho \dot{\psi} + \frac{1}{2} \rho (\nabla \psi)^2 + u(\rho) \right), \quad (28)$$

where dot means time derivative,  $\rho$  is fluid density,  $u$  is the internal energy density and because we are considering an irrotational flow  $\psi$  represent the velocity potential given by  $\vec{v} = \nabla \psi$ . Performing variations of the action (28) respect to  $\psi$  and  $\rho$  it is possible to obtain continuity and Bernoulli equations respectively. As we mentioned before, the basis of the analogy between gravitational black hole and sonic black holes comes from considering the propagation of acoustic disturbances ( $\bar{\rho}$  and  $\bar{\psi}$ ) on a barotropic, inviscid, inhomogeneous and irrotational (at least locally) fluid flow described by  $(\bar{\rho}_0$  and  $\bar{\psi}_0)$ . It is well known that the equation of motion for this acoustic disturbance (described by its velocity potential  $\bar{\psi}$ ) is identical to the Klein-Gordon equation for a massless scalar field minimally coupled to gravity in a curved space [39, 40, 41, 42]. For the disturbance the action (28) up to quadratic order becomes,

$$S = S_0 + S_2, \quad (29)$$

where  $S_2$  described the action of perturbations and it is given by

$$S_2 = - \int d^4x \left( \frac{1}{2} \rho (\nabla \bar{\psi})^2 - \frac{\rho}{2c^2} (\bar{\psi} + \vec{v}_0 \cdot \nabla \bar{\psi}) \right). \quad (30)$$

At this point we would like to note that this action can be written in more elegant form

$$S_2 = - \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \bar{\psi} \partial_\nu \bar{\psi}. \quad (31)$$

Therefore  $S_2$  is completely equivalent to the action for a massless scalar field (field that described sound perturbation) propagating in a curved space-time whose acoustic metric  $g^{\mu\nu}$  given by

$$g_{\mu\nu} = -\frac{\rho}{c} \begin{pmatrix} -1 & \cdots & -v_o^j \\ \vdots & \ddots & \vdots \\ -v_o^j & \cdots & c^2 \delta^{ij} - v_o^i v_o^j \end{pmatrix}, \quad (32)$$

where  $c$  represent the sound velocity<sup>1</sup>. In Unruh's work, the acoustic geometry is described by the following sonic line element

$$ds^2 = \frac{\rho_0}{c} \left( - (c^2 - v_o^2) d\tau^2 + \frac{c dr^2}{c^2 - v_o^2} + r^2 d\Omega^2 \right), \quad (33)$$

where  $\rho_0$  is the density of the fluid,  $c$  is the velocity of sound in the fluid (by simplicity we will assume these quantities constant) and  $v_o^r$  represent the radial component of the flow velocity. On the other hand, if then we assume that at some value of  $r = r_+$  we have the background fluid smoothly exceeding the velocity of sound,

$$v_o^r = -c + a(r - r_+) + \vartheta(r - r_+)^2, \quad (34)$$

the above metric assumes just the form it has for a Schwarzschild metric near the horizon. In this limit our metric reads as follows

$$ds^2 = \frac{\rho_0}{c} \left( -2ac(r - r_+)d\tau^2 + \frac{cdr^2}{2ac(r - r_+)} + r^2 d\Omega^2 \right), \quad (35)$$

where  $a$  is a parameter associated with the velocity of the fluid defined as  $(\nabla \cdot \vec{v})|_{r=r_+}$  [44]. Note that this geometry was study in Refs. [44, 45] where the authors studied the low energy dynamics and obtained the greybody factors for the sonic horizon from the absorption and the reflection coefficients, and the quasinormal modes respectively.

In the quantum version of this system we can hope that the acoustic black hole emits acoustic "Hawking radiation". This effect coming from the horizon of events is a pure kinematical effect that occurs in any Lorentzian geometry independent of its dynamical content [40]. It is well known that the acoustic metric does not satisfied the Einstein equations, due to the fact that the background fluid motion is governed by the continuity and the Euler equations. As a consequence of this fact, one should expect that the thermodynamic description of the acoustic black hole is ill defined. However, this powerful analogy between black hole physics and acoustic geometry admit to extend the study of many physical quantities and new method developed for describe black holes physics, such as Robinson-Wilczek anomaly gravitational approach, we are considering in the following the application of this methods in order to compute the Hawking temperature for this acoustic geometry. The action (31) for the scalar field  $\bar{\psi}$  in the background of dumb hole described before is given by

$$S(\bar{\psi}) = \int dr d\tau r^2 \int \sin\theta d\theta d\phi \bar{\psi} \left( \frac{1}{r^2} \partial_r (f(r) r^2 \partial_r) - \frac{1}{f(r)} \partial_\tau^2 + \frac{1}{r^2} \nabla_\Omega^2 \bar{\psi} \right),$$

where  $\nabla_\Omega^2$  represent the Laplacian operator on unitary two sphere. Passing the radial coordinate to tortoise coordinate ( $r^*$ ) whose transformation is defined by  $\frac{\partial r^*}{\partial r} = \frac{1}{f(r)}$  and performing the partial waves decomposition  $\bar{\psi} = \sum_l \bar{\psi}_l Y_l(\theta, \phi)$ , where  $l$  is the collection of angular quantum numbers and  $\bar{\psi}_l$  depends on the coordinates  $t$  and  $r$ . One found that the action near the horizon becomes

$$S(\bar{\psi}) = \sum_l \int d\tau dr r^2 \bar{\psi}_l \left[ -\frac{1}{f(r)} \partial_\tau^2 + \frac{1}{r^2} \partial_r (r^2 f(r) \partial_r) \right] \bar{\psi}_l. \quad (36)$$

Then, physics near the horizon can be described using an infinite collection of massless two-dimensional scalar fields in the following background

$$\begin{aligned} ds^2 &= -f(r) d\tau^2 + \frac{1}{f(r)} dr^2, \\ \phi &= r^2, \end{aligned} \quad (37)$$

where  $\phi$  is the two dimensional dilaton field and the flux is given by (24)

$$\Phi = N_t^r = \frac{1}{192\pi} f'^2(r_H) = \frac{\tilde{a}^2}{48\pi}. \quad (38)$$

Therefore applying the RW method, we get the following Hawking's temperature for the Unruh's dumb acoustic black hole

$$T_H = \frac{\tilde{a}}{2\pi}. \quad (39)$$

This result coincides with the well-known result of the Hawking temperature for this geometry [44] and the thermal emission is only proportional to the control parameter  $a$ .

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<sup>1</sup> Greek indices run from 0-3, while Roman indices run from 1-3

#### IV. CANONICAL ACOUSTIC BLACK HOLE

Another solution for the fluid flow is the called Canonical acoustic black hole. It was introduced by Visser in Ref. [40]. In the reference was found a solution with a spherically symmetric flow of incompressible fluid. This implies the density  $\rho$  is a position independent quantity and the continuity equation implies that  $v \sim 1/r^2$ . How was considered a barotropic equation of state pressure is also an independent position quantity. Therefore the speed of sound also is a constant. The metric for the canonical acoustic black hole is [40]

$$ds^2 = -c^2(1 - \frac{r_0^4}{r^4})dt^2 + (1 - \frac{r_0^4}{r^4})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (40)$$

where  $r_0 = \left(\frac{v r^2}{c}\right)^{1/2}$  is a normalization constant and without of lack we can consider  $c = 1$  and the metric (40) can be write as Schwarzschild type, as follow

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega_{(d-2)}^2, \quad (41)$$

where

$$f(r) = (1 - \frac{r_0^4}{r^4}), \quad (42)$$

physics properties as quasinormal modes of this kind of acoustic geometry were studied in Refs. [46] and [47]. In order to perform the method of Robinson-Wilczek we need to write the action (31) for the scalar field  $\bar{\psi}$  in the background of the Canonical acoustic black hole is

$$S(\bar{\psi}) = \int dr dtr^2 \int \sin\theta d\theta d\phi \bar{\psi} \left( \frac{1}{r^2} \partial_r (f(r) r^2 \partial_r) - \frac{1}{f(r)} \partial_t^2 + \frac{1}{r^2} \nabla_\Omega^2 \right) \bar{\psi}, \quad (43)$$

where  $\nabla_\Omega^2$  represent the Laplacian operator on unitary two sphere. Passing the radial coordinate to tortoise coordinate ( $r^*$ ) whose transformation is defined by  $\frac{\partial r^*}{\partial r} = \frac{1}{f(r)}$  and performing the partial waves decomposition  $\bar{\psi} = \sum_l \bar{\psi}_l Y_l(\theta, \phi)$ , where  $l$  is the collection of angular quantum numbers and  $\bar{\psi}_l$  depends on the coordinates  $t$  and  $r$ . One found that the action near the horizon becomes

$$S(\bar{\psi}) = \sum_l \int dt dr r^2 \bar{\psi}_l \left[ -\frac{1}{f(r)} \partial_t^2 + \frac{1}{r^2} \partial_r (r^2 f(r) \partial_r) \right] \bar{\psi}_l. \quad (44)$$

Therefore physics near the horizon can be described using an infinite collection of massless two-dimensional scalar fields in the following background

$$ds^2 = -f(r) dt^2 + \frac{1}{f(r)} dr^2, \quad (45)$$

and the flux is given by (24)

$$\Phi = N_t^r = \frac{1}{192\pi} f'^2(r_H) = \frac{1}{12\pi r_0^2}. \quad (46)$$

Therefore, we get the following Hawking's temperature for the canonical acoustic black hole

$$T_H = \frac{1}{\pi r_0}. \quad (47)$$

#### V. HAMILTON-JACOBI METHOD AND TUNNELING AT EVENT HORIZON

We will now review an alternative method for calculating black hole tunneling that makes use of the Hamilton-Jacobi equation as an ansatz [9]. This method ignores the effects of the particle self-gravitation and is based on the work of Padmanabhan and his collaborators [4, 5, 6]. In general the method involves using the WKB approximation to solve a wave equation. The simplest case to model is scalar particles, which therefore involves applying the WKB

approximation to the Klein-Gordon equation. The result, to the lowest order of WKB approximation, is a differential equation that can be solved by plugging in a suitable ansatz. The ansatz is chosen by using the symmetries of the spacetime to assume separability. After plugging in a suitable ansatz, the resulting equation can be solved by integrating along the classical forbidden trajectory, which starts inside the horizon and finishes at the outside observer. Since this trajectory is classically forbidden the equation will have a simple pole located at the horizon. So it is necessary to apply the method of complex path analysis and deflect the path around the pole. We will apply this method to the acoustics metrics previous. The Klein-Gordon equation for a scalar field  $\Phi$  is:

$$g^{\mu\nu}\partial_\mu I\partial_\nu\Phi - \frac{m^2}{\hbar^2}\Phi = 0. \quad (48)$$

Applying the WKB approximation by assuming an ansatz of the form

$$\Phi(t, r, x^i) = \exp\left[\frac{i}{\hbar}I(t, r, x^i)\right], \quad (49)$$

and then inserting this back into the Klein Gordon equation will result in the relativistic Hamilton-Jacobi equation to the lowest order in  $\hbar$

$$g^{\mu\nu}\partial_\mu I\partial_\nu I + m^2 = 0, \quad (50)$$

where  $m$  is the mass of the scalar particle. Considering Eq. (35), Eq. (50) can be rewritten as

$$-\frac{1}{f(r)}(\partial_t I)^2 + f(r)(\partial_r I)^2 + \frac{1}{r^2}(\partial_\theta I)^2 + \frac{1}{r^2 \sin^2 \theta}(\partial_\varphi I)^2 + m^2 = 0. \quad (51)$$

As usual, due to the symmetries of the metric, we can suppose a solution as following form

$$I = -Et + W(r) + J(x^i), \quad (52)$$

therefore we have

$$\partial_t I = -E, \partial_r I = W'(r), \partial_\theta I = J_\theta, \partial_\varphi I = J_\varphi, \quad (53)$$

where  $J_\theta$  and  $J_\varphi$  are constant respectively. Then, putting (53) into (51), we can get the classical action of an outgoing particle

$$I = -Et + \int \frac{\sqrt{E^2 - f(r)\left(\frac{1}{r^2}J_\theta^2 + \frac{1}{r^2 \sin^2 \theta}J_\varphi^2 + m^2\right)}dr}{f(r)} + J(x^i). \quad (54)$$

The metric coefficients for sector 'r-t' alter sign at the two sides of the event horizon. Therefore, the path in which tunneling takes place has an imaginary time coordinate. The ingoing and outgoing probabilities are now given by

$$P_{in} = |\Phi_{in}|^2 = \exp\left[-\frac{2}{\hbar}\left(-EImt - EIm \int_C \frac{dr}{f(r)}\right)\right], \quad (55)$$

and

$$P_{out} = |\Phi_{out}|^2 = \exp\left[-\frac{2}{\hbar}\left(-EImt + EIm \int_C \frac{dr}{f(r)}\right)\right]. \quad (56)$$

In the classical limit ( $\hbar \rightarrow 0$ ), we have

$$Imt = -Im \int_C \frac{dr}{f(r)}. \quad (57)$$

As a result the outgoing probability for the tunneling particle becomes,

$$P_{out} = |\Phi_{out}|^2 = \exp\left[-\frac{4}{\hbar}EIm \int_C \frac{dr}{f(r)}\right]. \quad (58)$$

The principle of "detailed balance" [4, 5, 6, 10] for ingoing and outgoing probabilities states that

$$P_{out} = \exp\left(-\frac{E}{T_H}\right) P_{in} = \exp\left(-\frac{E}{T_H}\right). \quad (59)$$

Comparing eq.(58) and eq.(59) we obtain the temperature for the Unruh's acoustic black hole, which is given by

$$T_H = \frac{\hbar}{4} \left( \text{Im} \int_C \frac{dr}{f(r)} \right)^{-1} \quad (60)$$

and using the expression  $f(r) = 2a(r - r_H)$  we obtain

$$T_H = \frac{a}{2\pi}, \quad (61)$$

which is the same result obtained previous, via Robinson-Wilczek method. Via the same procedure we can to obtain the Hawking temperature for Canonical acoustic black hole coinciding with the previous result (Robinson-Wilczek method).

## VI. CONCLUSIONS

In this paper, we have considered a quantum scalar fields in a Unruh's and Canonical acoustic black hole background. Using two different methods we computed Hawking temperature of two acoustic geometries, obtaining the same Hawking temperature. The Robinson and Wilczek method shown that near the horizon, the physics is described using an infinite collection of massless 1 + 1 dimensional scalar fields, knowing as phonons for a acoustic geometry. Also we show the robustness of the anomaly method for computed the Hawking temperature associated whit the even horizon of those acoustic geometry. Mainly, we showed that the Hawking radiation from Unruh's and Canonical acoustic black hole. In case of Unruh's black hole we found that the Hawking temperature coincides with the well-known result of the Hawking temperature for this geometry [44] and the thermal emission is only proportional to the control parameter  $a$ . We showed that the gravitational anomaly that appears in the Unruh's and Canonical acoustic black hole background is cancelled by the flux of a 1 + 1 dimensional blackbody radiation at the Hawking temperature. Certainly, the only way to probe that our fluxes are really Hawking fluxes, we need to compare they with the Hawking fluxes that one obtains from integrating the Planck distribution for a general charged rotating black hole. In order to avoid superradiance, we considered fermions in our descriptions. For fermions, the Planck distribution for blackbody radiation at the Hawking temperature  $T_H$ , is given by

$$N_{e,m}(\omega) = \frac{1}{e^{\omega - e\Phi - m\Omega_H} + 1}. \quad (62)$$

Then, Hawking fluxes can be computed by

$$F_M = \int_0^\infty \frac{d\omega}{2\pi} \omega (N_{e,m}(\omega) - N_{-e,-m}(\omega)), \quad (63)$$

where we are including the contribution from the antiparticles. Then from the evaluation of (63) for one uncharged and static geometric we obtain  $F_M = \frac{\pi}{12} T_H^2$ , that exactly match with (38). Therefore, we can concluded that the anomaly method and tunneling method are valid for the acoustic geometric. We believed that these methods can be useful for give a thermodynamics description of acoustics geometries despite that the dumb black holes do not satisfied Einstein's equation. We hope to discus this issue in the near future.

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